



EmSAT Achieve Math Public Test Specification

Test Description: EmSAT Achieve Math assesses the extent to which the test taker is ready to study mathematics at the college or university level. It is a computer-based exam where test sections, questions, and options are randomized. The tests are timed by the computer. Test-takers can see how much time they have throughout the exam.

This exam is adaptive. Exam content and difficulty is customized to the individual test taker. When a test taker answers a question correctly, they will be given more difficult content; when they answer a question incorrectly, they will be given easier content. This process of continuous adjustment delivers optimized content for each test taker throughout the exam, maximizing their opportunity to perform at their best and providing a more accurate measure of their ability.

Test Duration:	120 minutes
Questions:	50 questions
Content Areas:	Algebra, Geometry, and Statistics
Delivery Format	Computer Adaptive with Calculator Permitted
Task Types:	Multiple Choice and Fill-in the-Blank



EmSAT Achieve Math	
Score	Score Descriptors
1750 – 2000	A student at this level has demonstrated sufficient mastery of higher-level algebra and geometry, and basic calculus, to be considered for entry into a second calculus course.
1500 – 1725	A student at this level has demonstrated sufficient mastery of algebra and geometry to warrant being considered for direct entry into a university-level calculus course.
1100 – 1475	A student at this level has demonstrated sufficient understanding of algebra and geometry to be able to engage in pre-calculus or similar courses of bachelor level instruction. Additional preparation would be recommended for those seeking a STEM major.
700 – 1075	A student at this level has demonstrated some understanding of algebra and applied geometry. A student at this level should consider enrolling in mathematics preparation courses before enrolling in year one bachelor level mathematics courses.
500 – 675	A student at this level has demonstrated some numerical understanding but a very limited understanding of basic algebra and geometry concepts. This student needs additional instruction and support in basic numeracy, algebra and geometry prior to engaging in tertiary level mathematics instruction.
< 500	A student at this level has demonstrated little or no ability to apply basic mathematical concepts at tertiary entrance level. Students in this level would need to significantly improve their mathematical understanding before engaging in any tertiary level mathematics instruction.



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Appendix 1: Content Areas

Content Area 1: Algebra

- Interpret the structure of expressions
 - Write expressions in equivalent forms to solve problems
 - Perform arithmetic operations on polynomials
 - Understand the relationship between zeros and factors of polynomials
 - Use polynomial identities to solve problems
 - Rewrite rational functions
 - Create equations that describe numbers or relationships
 - Understand solving equations as a process of reasoning and explain the reasoning
 - Solve equations and inequalities in one variable
 - Solve systems of equations
 - Represent and solve equations and inequalities graphically
 - Understand the concept of a function and use function notation
 - Interpret functions that arise in applications in terms of the context
 - Analyze functions using different representations
 - Build a function that models a relationship between two quantities
 - Build new functions from existing functions
 - Construct and compare linear and exponential models and solve problems
 - Interpret expressions for functions in terms of the situation they model
 - Extend the domain of trigonometric functions using the unit circle
 - Model periodic phenomena with trigonometric functions
 - Prove and apply trigonometric identities
 - Extend the properties of exponents to rational exponents
 - Use properties of rational and irrational numbers.
 - Reason quantitatively and use units to solve problems
 - Perform operations with complex numbers
 - Represent and model with vector quantities.
 - Perform operations on vectors.
 - Perform operations on matrices and use matrices in applications
 - Solve problems with limits
 - Solve basic differentiation problems
 - Solve basic integration problems
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Appendix 1: Content Areas

Content Area 2: Geometry

- Transformations for 2D and 3D shapes
 - Understand congruence in terms of transformation
 - Prove geometric theorems
 - Understand similarity in terms of similarity transformations
 - Prove theorems involving similarity
 - Define trigonometric ratios and solve problems involving right triangles
 - Apply trigonometry to general triangles
 - Understand and apply theorems about circles
 - Translate between the geometric description and the equation for a conic section
 - Use coordinates to prove simple geometric theorems algebraically
 - Explain volume formulas and use them to solve problems
 - Visualize relationships between two-dimensional and three-dimensional objects
-

Content Area 3: Statistics

- Summarize, represent, and interpret data on a single variable
 - Summarize, represent, and interpret data on two categorical and quantitative variables
 - Interpret linear models
 - Understand and evaluate random processes underlying statistical experiments
 - Make inferences and justify conclusions from sample surveys, experiments and observational studies
 - Understand independence and conditional probability and use them to interpret data
 - Use the rules of probability to compute probabilities of compound events in a uniform probability model
 - Calculate expected values and use them to solve problems
 - Use probability to evaluate outcomes of decisions
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Appendix 2: Sample Items

1. What is the solution set of the the following equation?

ماهي مجموعة الحل للمعادلة التالية؟

$$4x^2 + 4x = 2^{-6}$$

- A.
- B.
- C.
- D.

2. Identify the quadrant when the sum of the complex numbers $3 + 2i$ and $6 - 4i$ is graphed.

في أي ربع بياني سنرسم حاصل جمع الأعداد

المركبة $3 + 2i$ و $6 - 4i$ ؟

- A.
- B.
- C.
- D.



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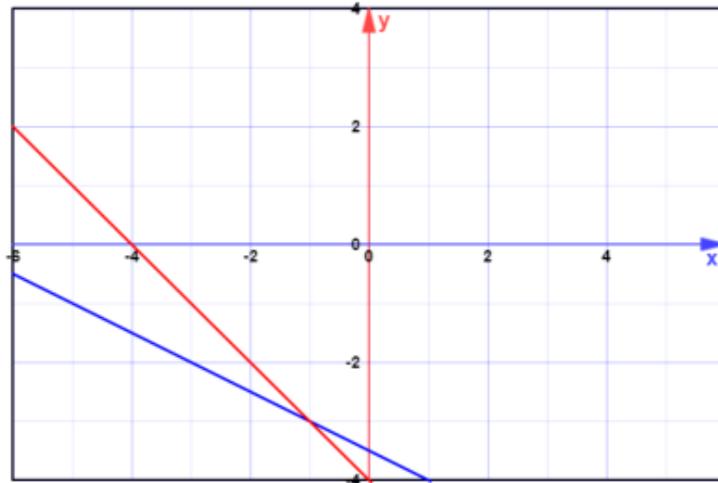
Appendix 2: Sample Items

3. The graphs for $L1$ and $L2$ are shown below.

يوضح الشكل أدناه الرسم البياني لـ $L1$ و $L2$

How is this system of equations classified?

كيف يتم تصنيف المعادلات في هذا النظام؟



- A. Consistent independent ثابت مستقل
- B. Inconsistent متعارض
- C. Consistent dependent ثابت تابع



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Appendix 2: Sample Items

4. The heights of boys in a grade 10 class are normally distributed with a mean of 168 cm and a standard deviation of 2.5 cm.

يعتبر توزيع أطوال الطلاب في الصف 10 توزيعاً معيارياً، بمتوسط مقداره 168 سم وانحراف معياري 2.5 سم.

In which range do 95% of the heights approximately fall?

ما هو المدى الذي تقع فيها 95% من الأطوال تقريباً؟

- A. 163 - 173 cm
- B. 160.5 - 168 cm
- C. 160.5 - 175.5 cm
- D. 163 - 175.5 cm

5. Simplify.

بَسِّط.

$$i^{37}$$

- A. i
- B. -1
- C. 1
- D. $-i$



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Appendix 2: Sample Items

6. Line segment $\overline{A'B'}$ has endpoints $A'(4, -2)$, $B'(16, 14)$ and is the image of \overline{AB} after a dilation of $\frac{1}{2}$ centered at the origin.

What is the length of \overline{AB} ?

القطعة المستقيمة $\overline{A'B'}$ طرفيها $A'(4, -2)$, $B'(16, 14)$ وهي صورة للقطعة المستقيمة \overline{AB} بعد تمدد معاملته $\frac{1}{2}$ حول نقطة الأصل.

ما هو طول \overline{AB} ؟

- A.
- B.
- C.
- D.



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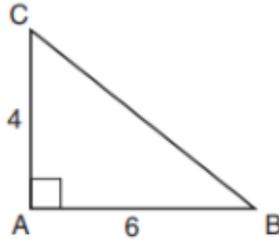
Appendix 2: Sample Items

7. In the diagram below, right triangle ABC has legs whose lengths are 4 and 6.

في الرسم البياني أدناه، مثلث قائم الزاوية ABC طول ضلعيه 4 و 6.

What is the volume of the three-dimensional object formed by continuously rotating the right triangle about AB ?

ما هو المجسم الثلاثي الأبعاد الناتج من الدوران المستمر للمثلث حول الضلع AB ؟



- A.
- B.
- C.
- D.



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Appendix 2: Sample Items

8. Write the expression below in simplest form.

اكتب التعبير أدناه بأبسط صورة

$$\sqrt{-300}$$

A.

$$10i\sqrt{3}$$

B.

$$3i\sqrt{10}$$

C.

$$5i\sqrt{12}$$

D.

$$12i\sqrt{5}$$

9. Select **all** value(s) for which m in the following expression defined?

اختر كل القيم من العبارة التالية التي تعتبر فيها قيمة m غير معرفة؟

For this item type you can select 1, 2, 3, or all choices.

لهذه السؤال ، يمكن اختيار 1,2,3 أو كل الخيارات

$$\frac{15m^2n}{3-m}$$

A.

$$-3$$

B.

$$0$$

C.

$$1$$

D.

$$3$$



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Appendix 2: Sample Items

10.

Suppose that $\left(-\frac{\sqrt{7}}{3}, y\right)$ is a point in quadrant II lying on the unit circle.

Find y .

في دائرة الوحدة، إذا كانت النقطة $\left(-\frac{\sqrt{7}}{3}, y\right)$ تقع في الربع الثاني.

فأوجد y .

A.

$$\frac{\sqrt{2}}{3}$$

B.

$$\frac{\sqrt{6}}{3}$$

C.

$$-\frac{\sqrt{3}}{3}$$

D.

$$\frac{\sqrt{7}}{3}$$



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Appendix 2: Sample Items

11. Select **all** true statements.

اختر كل العبارات الصحيحة.

- A. Since 13.1 is a terminating decimal, it is **rational**.
بما أن 13.1 عدد عشري منتهي، فهو عدد نسبي
- B. Since $\frac{14}{4}$ is a ratio of two integers, it is **rational**.
بما أن $\frac{14}{4}$ تناسب لعددين صحيحين، فهو عدد نسبي
- C. Since 7 is **not** a perfect square, $\sqrt{7}$ is **rational**.
إبما أن 7 ليس مربعاً كاملاً، فإن $\sqrt{7}$ يعتبر عدد نسبي
- D. Since 16 is a perfect square, $\sqrt{16}$ is **irrational**.
بما أن 16 مربعاً كاملاً، فإن $\sqrt{16}$ يعتبر عدد غير نسبي
- E. Since $5.\overline{84}$ is a repeating decimal, it is **rational**.
إبما أن $5.\overline{84}$ عدد عشري دوري، فهو عدد نسبي.



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Appendix 2: Sample Items

12. What is period of the graph of the function below?

ماهي دورة الدالة أدناه؟

$$y = -\frac{2}{3} \sin\left(\frac{3}{2}x\right)$$

A.

$$\frac{4}{3\pi}$$

B.

$$4\pi$$

C.

$$\frac{2\pi}{3}$$

D.

$$2\pi$$



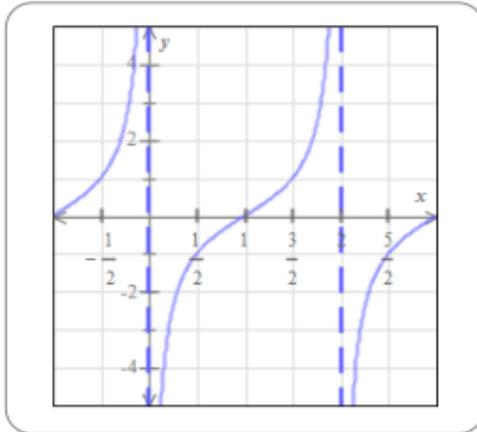
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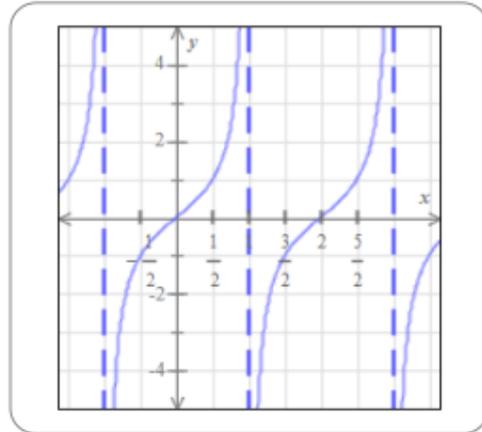
13. Which graph represents the function
 $y = -\cot\left(\frac{\pi x}{2}\right)$?

أي من الرسوم البيانية التالية تمثل الدالة
 $y = -\cot\left(\frac{\pi x}{2}\right)$ ؟

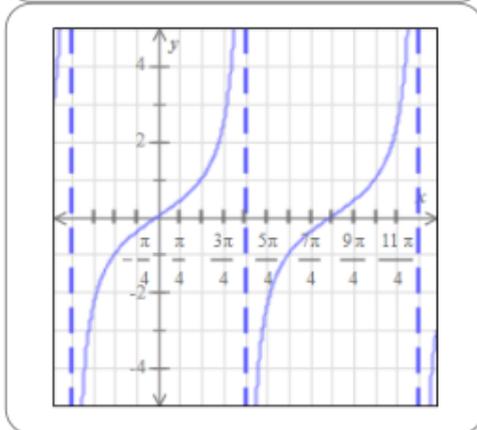
A.



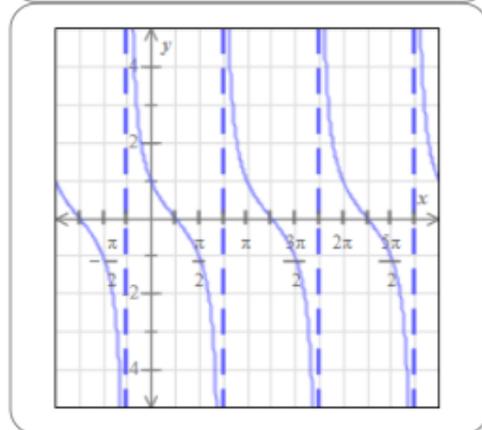
B.



C.



D.





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Appendix 2: Sample Items

14. What is the sum of $\sqrt{27}$ and $\sqrt{108}$?

ما هو حاصل جمع $\sqrt{27}$ و $\sqrt{108}$ ؟

A.

$$9\sqrt{3}$$

B.

$$\sqrt{135}$$

C.

$$3\sqrt{3}$$

D.

$$4\sqrt{27}$$



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Appendix 2: Sample Items

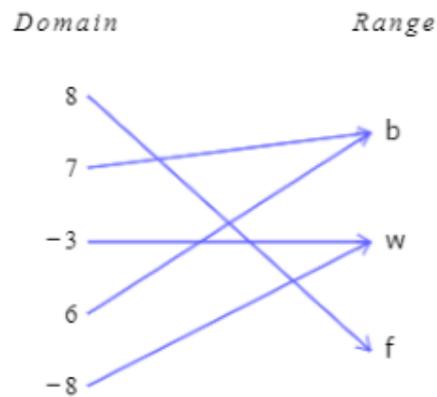
15. Select **all** relations that are a function.

اختر جميع العلاقات التي تمثل دوالاً.

A.

$\{(9, 5), (2, 5), (5, 5), (0, 5)\}$

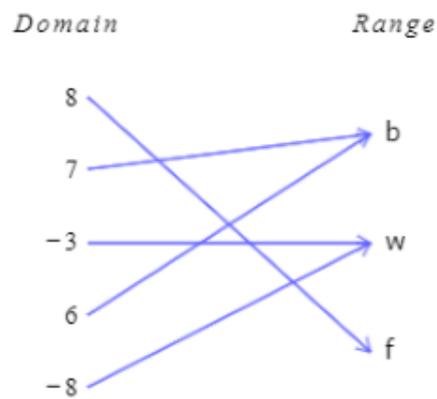
B.



C.

$\{(3, 2), (3, 3), (3, -4), (3, -1)\}$

D.





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Appendix 2: Sample Items

16. Identify the image of point $(4, -2)$ after a dilation of 3. عرف صورة الأزواج المرتبة $(4, -2)$ بعد التمدد بـ 3.

- A. $(12, -6)$
- B. $(\frac{4}{3}, -\frac{2}{3})$
- C. $(7, 1)$
- D. $(1, -5)$

17. Solve. حل المعادلة التالية :

$$y^2 - 3y = 9$$

- A. $\frac{3 \pm 3\sqrt{5}}{2}$
- B. $\frac{-3 \pm 3\sqrt{5}}{2}$
- C. $\frac{3 \pm 3i\sqrt{5}}{2}$
- D. $\frac{3 \pm 3i\sqrt{3}}{2}$



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Appendix 2: Sample Items

18. Choose the correct classification for the conic given below.

اختر التصنيف الصحيح للشكل الذي تمثله المعادلة أدناه.

$$2y^2 + 12y - x + 17 = 0$$

- A. Parabola قطع مكافئ
- B. Hyperbola قطع زائد
- C. Circle دائرة
- D. Ellipse قطع ناقص

19. Which equation represents a circle whose center is $(3, -1)$ and whose radius is $\sqrt{6}$?

أي المعادلات التالية توضح دائرة مركزها $(3, -1)$ ونصف قطرها $\sqrt{6}$ ؟

- A. $(x - 3)^2 + (y + 1)^2 = 6$
- B. $(x + 3)^2 + (y - 1)^2 = 6$
- C. $(x - 3)^2 + (y + 1)^2 = 36$
- D. $(x + 3)^2 + (y - 1)^2 = 36$



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Appendix 2: Sample Items

20. If $m\angle A = 35$, $b = 3$, and $a = 4$, how many different triangles can be constructed?

إذا كان $m\angle A = 35$, $b = 3$, $a = 4$ كم عدد المثلثات المختلفة التي يمكن رسمها؟

- A. one obtuse triangle, only مثلث واحد فقط منفرج الزاوية
- B. one right triangle, only مثلث واحد فقط قائم الزاوية
- C. two triangles مثلثان
- D. no triangles can be constructed لا يمكن بناء أي مثلث



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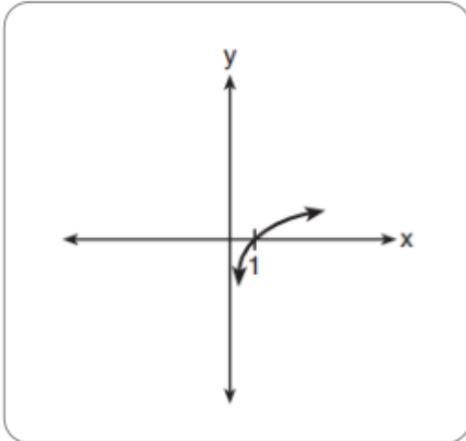
Appendix 2: Sample Items

21. Which sketch shows the inverse of the equation given below?

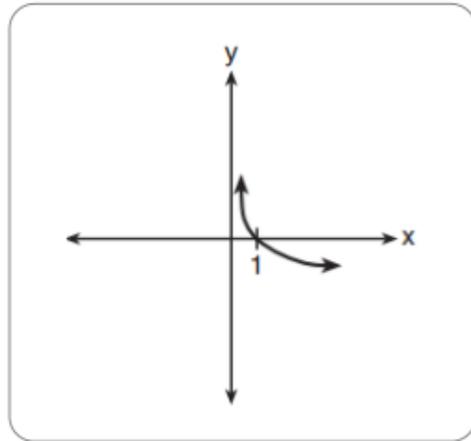
أي رسم يمثل المعادلة العكسية للمعادلة أدناه؟

$$y = a^x, a > 1$$

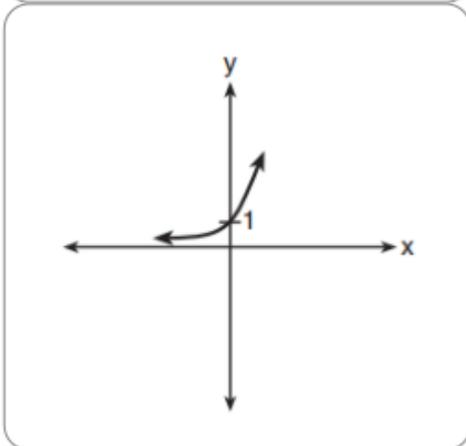
A.



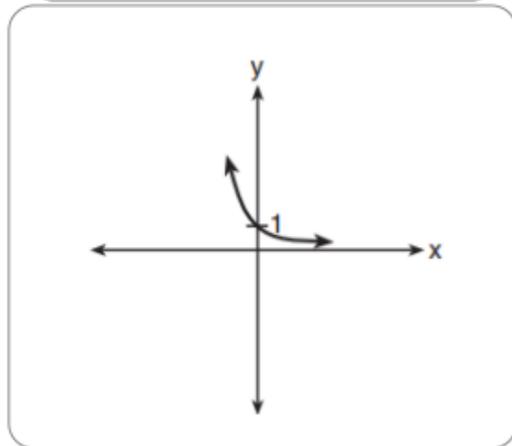
B.



C.



D.





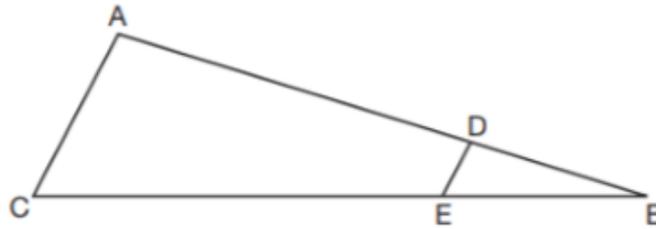
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Appendix 2: Sample Items

22. What is the length of \overline{AC} ?

ما هو طول الضلع \overline{AC} ؟

$$\overline{AC} \parallel \overline{DE}, AD = 24, DB = 12, DE = 4$$



- A.
- B.
- C.
- D.



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Appendix 2: Sample Items

23. Aisha calculated the area of a piece of paper to be 50.27 cm^2 .

حسبت عائشة مساحة قطعة من الورق أنها
 50.27 cm^2

The actual area of the paper is
 55.42 cm^2 .

إلا أن المساحة الحقيقية لهذه الورقة هي
 55.42 cm^2

What is the margin of error in calculating
the area?

ماهي درجة الانحراف النسبي في حساب هذه
المساحة؟

- A.
- B.
- C.
- D.

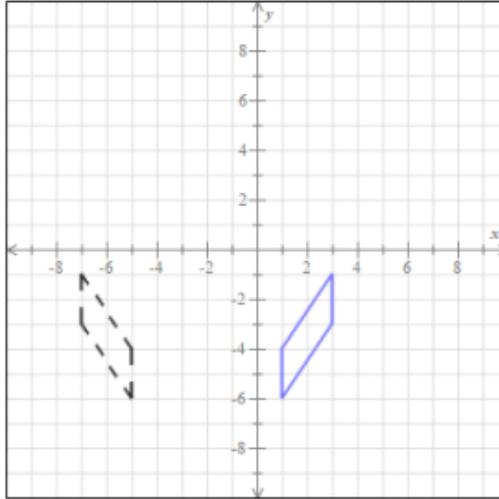


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Appendix 2: Sample Items

24. How are these figures related?

ما العلاقة بين الشكلين أدناه؟



- | | | |
|----|-------------|----------------|
| A. | Reflection | إنعكاس |
| B. | Rotation | دوران |
| C. | Translation | إزاحة |
| D. | Not related | لا توجد علاقة |
| E. | Dilation | تكبير أو تصغير |



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Appendix 2: Sample Items

25. Simplify.

بسط

$$\frac{y-x}{x^2-y^2}$$

A.

$$\frac{-1}{x+y}$$

B.

$$\frac{1}{x+y}$$

C.

$$\frac{-1}{x-y}$$

D.

$$\frac{1}{x-y}$$



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Appendix 2: Sample Items

26.

A frequency table of temperatures is created from an experiment in science class.

تم تكوين جدول تكراري لدرجات الحرارة من تجربة في صف العلوم.

Interval	0-9	10-19	20-29	30-39	الفترة (الحرارة)
Frequency	7	5	6	13	التكرار

Which interval contains the median temperature?

ما هي الفترة التي تحتوي على وسيط درجة الحرارة؟

A.

20 – 29

B.

30 – 39

C.

10 – 19

D.

0 – 9



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Appendix 2: Sample Items

27. Which rotation about its center will map a regular decagon onto itself?

لدينا مضلع منتظم ذو عشرة أضلاع.

أي دوران حول نقطة المركز ينتج عنه مضلع ينطبق على نفسه؟

- A.
- B.
- C.
- D.



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Appendix 2: Sample Items

28.

Ahmed and Hamad play tennis each week.

يلعب أحمد وحمد كرة التنس كل أسبوع.

The probability that Ahmed wins the first match against Hamad is $\frac{2}{3}$.

احتمالية أن يفوز أحمد المباراة الأولى ضد حمد هو $\frac{2}{3}$ (مباراتان من أصل ثلاث).

What is the probability that Ahmed wins **exactly** three of the next four matches against Hamad?

ماهي احتمالية ان يفوز أحمد ثلاث مباريات بالضبط في الأربع مباريات القادمة ضد حمد؟

A.

$$\frac{32}{81}$$

B.

$$\frac{8}{81}$$

C.

$$\frac{16}{243}$$

D.

$$\frac{32}{243}$$



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Appendix 2: Sample Items

29. Find a solution to the following system of equations.

أوجد مجموعة الحل للمعادلات التالية.

$$x^2 + y^2 = 7$$

$$x^2 - y = 5$$

A.

$$(x, y) = (\sqrt{3}, 2)$$

B.

$$(x, y) = \left(-\frac{4}{5}, \frac{116}{25}\right)$$

C.

$$(x, y) = (5, 177)$$

D.

No solution لا يوجد حل

30. Solve for x.

أحل المعادلة التالية

$$\frac{2}{x+3} - \frac{3}{4-3} = \frac{2x-2}{x^2-x-12}$$

A.

-1

B.

-5

C.

all real numbers

جميع الأعداد الحقيقية

D.

no real solution

لا يوجد حل في الأعداد الحقيقية



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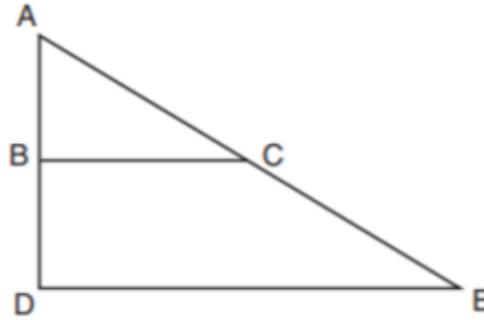
Appendix 2: Sample Items

31. $\triangle ADE$ is the image of $\triangle ABC$ after a dilation of scale factor k centered at the origin.

$\triangle ADE$ هو صورة للمثلث $\triangle ABC$ بعد تمدد بمعامل k حول نقطة الأصل.

Which statement is always true?

أي عبارة تكون صحيحة دائماً؟



A.

$$\overline{BC} \parallel \overline{DE}$$

B.

$$AC = CE$$

C.

$$\overline{AD} \perp \overline{DE}$$

D.

$$2AB = AD$$



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Appendix 2: Sample Items

32.

There is a $\frac{3}{5}$ probability of rain on Sunday.

The probability that it will rain on both Sunday and Monday is $\frac{3}{10}$.

If it rains on Sunday, what is the probability that it will rain on Monday?

هناك احتمالية $\frac{3}{5}$ لسقوط المطر يوم الأحد.

واحتمالية سقوطه يومى الأحد والاثنين هي $\frac{3}{10}$.

إذا كانت ستمطر يوم الأحد، ما احتمالية أنها ستمطر يوم الاثنين؟

A.

$$\frac{1}{2}$$

B.

$$\frac{3}{10}$$

C.

$$\frac{9}{50}$$

D.

$$\frac{9}{10}$$



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Appendix 2: Sample Items

33. A rhombus has diagonals with lengths 16 and 30.

لدى معين أقطار أطوالها 16 و 30

What is the length of a side of the rhombus?

ما هو طول أحد أضلاع المعين؟

- A.
- B.
- C.
- D.

34. Identify the **statistical data type** for the following variable: a medal won at the Olympics (gold, silver, bronze, or none).

حدد المعيار الاحصائي للمتغير التالي: ميدالية تم الفوز بها بالأولمبية (الذهبي، الفضي، البرونزي، لا شيء).

- A. إسمي
- B. معيار ترتيبي
- C. فترات
- D. نسب



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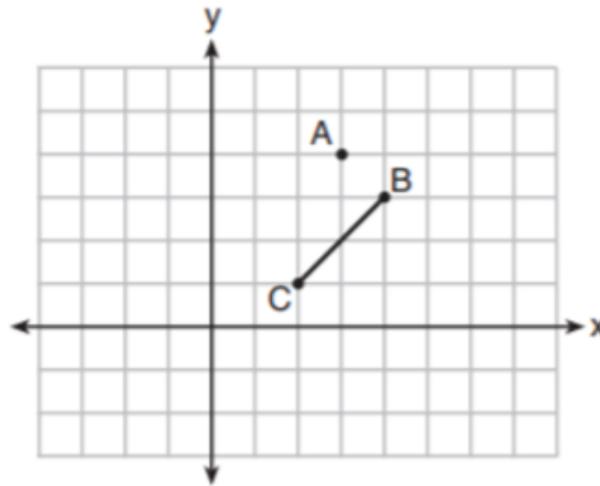
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35. Points $A(3, 4)$, $B(4, 3)$, $C(2, 1)$ are graphed below.

النقاط $A(3, 4)$, $B(4, 3)$, $C(2, 1)$ ، تم رسمها
بيانياً أدناه.

What are the coordinates of B' and C'
after \overline{BC} undergoes a dilation centered
at point A with scale factor of 2?

ماهي إحداثيات B' و C' بعد تمدد \overline{BC} حول
النقطة A بمعامل تكبير 2؟



A.

$$B'(5, 2) \text{ \& } C'(1, -2)$$

B.

$$B'(6, 1) \text{ \& } C'(0, -1)$$

C.

$$B'(5, 0) \text{ \& } C'(1, -2)$$

D.

$$B'(5, 2) \text{ \& } C'(3, 0)$$



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Appendix 2: Sample Items

Key:

1. A.
2. A.
3. A.
4. A.
5. A.
6. A.
7. A.
8. A.
9. A. B. C.
10. A.
11. A. B.
12. A.
13. A.
14. A.
15. A. B.
16. A.
17. A.
18. A.
19. A.
20. A.
21. A.
22. A.
23. A.
24. A.
25. A.
26. A.
27. A.
28. A.
29. A.
30. A.
31. A.
32. A.
33. A.
34. A.
35. A.



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Appendix 3: Formulas



Perimeter (P) and Circumference (C):

Quantity	Formula
Perimeter of a square	$P = 4d$
Perimeter of a quadrilateral	$P = s_1 + s_2 + s_3 + s_4$
Perimeter of a triangle	$P = s_1 + s_2 + s_3$
Perimeter of Trapezoid	$P = \frac{h(b_1 + b_2)}{2}$
Circumference of a circle	$C = 2\pi r$

Volume (V):

Quantity	Formula
Volume of a cube	$V = s^3$
Volume of a rectangular prism	$V = lwh$
Volume of a right cylinder	$V = \pi r^2 h$
Volume of a sphere	$V = \frac{4}{3} \pi r^3$
Volume of a cone	$V = \pi r^2 \frac{h}{3}$
Volume of a rectangular pyramid	$V = \frac{lwh}{3}$
Volume of a triangular prism	$V = \frac{bh l}{2}$

Area (A):

Quantity	Formula
Area of square	$A = s^2$
Area of a rectangle	$A = l \times w$
Area of a circle	$A = \pi r^2$
Area of a triangle	$A = \frac{bh}{2}$
Area of a parallelogram	$A = bh$
Area of trapezoid	$A = \frac{a + b}{2} h$
Area of a rhombus	$A = \frac{pq}{2}$
Area of Ellipse	$A = \pi r_1 r_2$
Area of Cube	$A = 6s^2$

Surface area (SA):

Quantity	Formula
Surface area of rectangular prism	$SA = 2(wl + hl + hw)$
Surface area of a cube	$SA = 6s^2$
Surface area of cylinder	$SA = 2\pi r h + 2\pi r^2$
Surface area of a sphere	$SA = 4\pi r^2$
Surface area of cone	$SA = \pi r s$

Units Conversion:

1 cm = 10 mm	1 g = 1000 mg	1 mL = 1 cm ³	K = °C + 273.15	1 psi = 0.068 atm
1 m = 100 cm	1 kg = 1000 g	1 dL = 100 mL	°C = (F - 32) × 5/9	1 atm = 101.325 kPa
1 m = 1000 mm	1 mg = 1000 µg	1 L = 10 dL	1 cal = 4.184 J	1 atm = 760 mmHg
1 km = 1000 m	1 lb = 16 oz	1 L = 1000 mL		1 atm = 1.01325 bar
1 ft = 12 in	1 kg = 2.20 lb	1 pint = 2 cups		1 mmHg = 1 torr
1 yard = 3 ft	454 g = 1 lb	1 qt = 4 cups		
1 mile = 5280 ft	1 ton = 907.2 kg	1 gallon = 4 qts		
1 in = 2.54 cm		946 mL = 1 qt		
1 yd = 0.914 m		1 L = 1.06 qt		
1 km = 0.621 miles				



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Appendix 3: Formulas



Logarithms:

1	$x = \log_b A \leftrightarrow b^x = A$
2	$\log x = \log_{10} x$
3	$\ln x = \log_e x$
4	$\ln x = \log_e x$
5	$\log_b (AB) = \log_b A + \log_b B$
6	$\log_b \left(\frac{A}{B}\right) = \log_b A - \log_b B$
7	$\log_b A^p = p \log_b A$

Exponential Equation:

$$A = A_0 e^{rt}$$

r is the annual growth/decay rate; $r < 0$, decay $r > 0$, growth

t is time in years

A_0 is amount present initially (present value)

A is the target value (future value)

Compound Interest Equations:

Compound interest: $A = P \left(1 + \frac{r}{n}\right)^{nt}$

Continuous compound interest: $A = Pe^{rt}$

P = present value

A = future value

r = annual interest rate

t = time in years

n = frequency of compounding per year



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Appendix 3: Formulas



Vectors:

$$\mathbf{v} = a\mathbf{i} + b\mathbf{j}$$

\mathbf{i} is the terminal point (1, 0)

\mathbf{j} is the terminal point (0, 1)

\mathbf{u} has initial point (x_1, y_1) and terminal point (x_2, y_2)

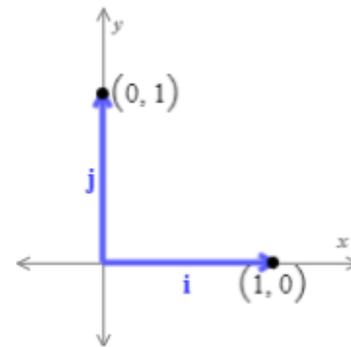
$$\mathbf{u} = (x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j}$$

\mathbf{v} has initial point (x_1, y_1) and terminal point (x_2, y_2)

$$\mathbf{v} = \langle x_2 - x_1, y_2 - y_1 \rangle$$

magnitude of a vector $\mathbf{v} = a\mathbf{i} + b\mathbf{j}$ is given by the formula

$$|\mathbf{v}| = \sqrt{a^2 + b^2}$$



Dot product vectors:

$$\mathbf{u} \cdot \mathbf{v} = a_1a_2 + b_1b_2$$

$$\mathbf{u} = a_1\mathbf{i} + b_1\mathbf{j} \text{ and } \mathbf{v} = a_2\mathbf{i} + b_2\mathbf{j}$$

$$\mathbf{u} = \langle a_1, b_1 \rangle \text{ and } \mathbf{v} = \langle a_2, b_2 \rangle$$



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Appendix 3: Formulas



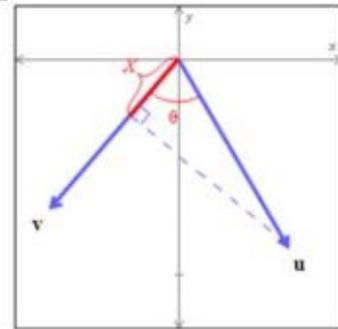
Angle between vectors:

For any two nonzero vectors u and v , the angle θ between them is given by

$$\cos \theta = \frac{u \cdot v}{|u||v|}$$

Component of a vector along a vector:

For any two nonzero vectors u and v , the angle θ between them is given by $\cos \theta = \frac{u \cdot v}{|u||v|}$



Complex Numbers:

$$i = \sqrt{-1}$$



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Appendix 3: Formulas



Limit Theorems:

<i>If $\lim_{x \rightarrow a} f(x)$ and $\lim_{x \rightarrow a} g(x)$ exist, then we have the following:</i>	
1	$\lim_{x \rightarrow a} c = c$ for any constant c
2	$\lim_{x \rightarrow a} x = a$
3	$\lim_{x \rightarrow a} [f(x) + g(x)] = \lim_{x \rightarrow a} f(x) + \lim_{x \rightarrow a} g(x)$
4	$\lim_{x \rightarrow a} [f(x) - g(x)] = \lim_{x \rightarrow a} f(x) - \lim_{x \rightarrow a} g(x)$
5	$\lim_{x \rightarrow a} cf = c \lim_{x \rightarrow a} f(x)$ for any constant c
6	$\lim_{x \rightarrow a} [f(x)g(x)] = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} g(x)$
7	$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)}$, as long as $\lim_{x \rightarrow a} g(x) \neq 0$
8	$\lim_{x \rightarrow a} [f(x)]^n = \left[\lim_{x \rightarrow a} f(x) \right]^n$ for any positive integer n
9	$\lim_{x \rightarrow a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \rightarrow a} f(x)}$ for any positive integer n . (if n is even, then $\lim_{x \rightarrow a} f(x)$ must be positive.)



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Appendix 3: Formulas



SERIES:

Aritmetic	$a_n = a_1 + (n - 1) \cdot d$	$S_n = \frac{n(a_1 + a_n)}{2}$
Geometric	$a_n = a_1 \cdot r^{n-1}$	$S_n = a_1 \frac{(1 - r^n)}{1 - r}$
Bionomial	$(a + b)^n = {}_n C_0 a^n b^0 + {}_n C_1 a^{n-1} b^1 + {}_n C_2 a^{n-2} b^2 + \dots + {}_n C_n a^0 b^n$ ${}_n C_r = \frac{n!}{r!(n-r)!}$ for $r = 0, 1, 2, \dots, n$.	$(a + b)^n = \sum_{r=0}^n {}_n C_r a^{n-r} b^r$

Conics:

Distance	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	
Midpoint	$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$	
Circle	$(x - h)^2 + (y - k)^2 = r^2$	
	Horizontal Major Axis	Vertical Major Axis
Parabola	$(x - h)^2 = 4p(y - k)$ Vertex: (h, k) Focus: (h, k+p) Directrix: y=k - p	$(y - k)^2 = 4p(x - h)$ Vertex: (h,k) Focus: (h+p, K) Directrix: x=h - p
Ellipse	Standard form: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ Center: (h, k) Vertices: $(h \pm c, k)$ where $c^2 = a^2 + b^2$ Endpoints of minor axis: $(h, k \pm b)$ Foci: $(h \pm c, k)$	Standard form: $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$ Center: (h, k) Vertices: $(h, k \pm c)$ where $c^2 = a^2 + b^2$ Endpoints of minor axis: $(h \pm b, k)$ Foci: $(h, k \pm c)$
Hyperbola	Standard form: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ Center: (h, k) Vertices: $(h \pm a, k)$ Foci: $(h, k \pm c)$ where $c^2 = a^2 + b^2$ Transverse axis: y = k Asymptotes: $y = \pm \frac{b}{a}(x - h) + k$	Standard form: $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$ Center: (h, k) Vertices: $(h, k \pm a)$ Foci: $(h, k \pm c)$ where $c^2 = a^2 + b^2$ Transverse axis: x = h Asymptotes: $y = \pm \frac{a}{b}(x - h) + k$

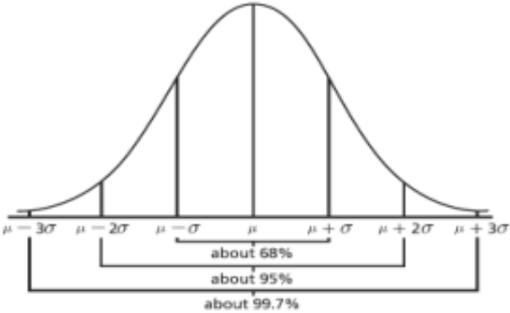


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Appendix 3: Formulas



Statistics:

Normal Curve Distribution	
Standard Deviation	<p>The standard deviation, σ, for values $x_1, x_2, x_3, \dots, x_n$ with mean μ is determined by the following:</p> $\sigma = \sqrt{\frac{(x_1 - \mu)^2 + (x_2 - \mu)^2 + \dots + (x_n - \mu)^2}{n}}$
Correlation Coefficient	$r = \frac{n \sum xy - (\sum x)(\sum y)}{\sqrt{[n \sum x^2 - (\sum x)^2][n \sum y^2 - (\sum y)^2]}}$ <p> n = number of pairs of scores $\sum xy$ = sum of the products of paired scores $\sum x$ = sum of x scores $\sum y$ = sum of y scores $\sum x^2$ = sum of squared x scores $\sum y^2$ = sum of squared y scores </p>



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Appendix 3: Formulas



Cramer's Rule for Solving a System of Linear Equations:

For a 2×2 System:

$$\begin{aligned} a_1x + b_1y &= c_1 \\ a_2x + b_2y &= c_2 \end{aligned} \quad x = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} \quad y = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

For a 3×3 System:

$$\begin{aligned} a_1x + b_1y + c_1z &= d_1 \\ a_2x + b_2y + c_2z &= d_2 \\ a_3x + b_3y + c_3z &= d_3 \end{aligned} \quad x = \frac{\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}} \quad y = \frac{\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}} \quad z = \frac{\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}}$$